# In Search of a New Space of K-12 Mathematics in the Classroom

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Abstract—When proposing structural changes in an educational system three elements are required: a set of oppositional principles to evaluate alternatives; a formal analysis of the status quo; and a comprehensive description and evaluation of proposed spaces. Using this three-phase approach, this paper proposes a set of alternative spaces of mathematics for K-12 education in which the skills demanded by today's society are taken into consideration. In addition, these spaces of mathematics supplement the traditional and limited fields of arithmetic, Euclidean and Cartesian geometries, algebra, and basic trigonometry. The cognitive capabilities and limitations of the human mind that in the last two decades have been identified are described and illustrated as guide for the evaluation of oppositional principles. This paper formally evaluates both the cognitive demands and the potentials for growth and development of areas in mathematics in current and alternative educational spaces, and uses specific educational examples to illustrate the mathematical, pedagogical and cognitive potentials and limitations of current and alternative mathematical educational systems.

*Index Terms*—mathematics education, human cognition, descriptive and generative languages, curriculum, oppositional principles

# I. INTRODUCTION

The present environment of mathematics education around the world is far from being a rich field for curricular and pedagogical survey where new content is explored, new pedagogical approaches are introduced, and the new cognitive tasks that modern society demands are incorporated. Rather, it has become a monolithic entity where global standards are imposed, obsolete content is preserved and reinforced, and failed pedagogical strategies are perpetuated [1]-[3]

This status quo is maintained by the significant influence exerted by the global educational standards that the developed world, and by implication the developing world, has set for itself in the last two decades [4]-[6] The group of developed countries of the world, under the organizational umbrella of the OECD (Organization for Economic Cooperation and Development), as well as the PISA (Programme for International Student Assessment) exams, evaluations and recommendations, have created a system by which the educational systems of the world are evaluated under a common standard. In addition, the programs of the most successful countries are identified, and those are set as references for other countries to emulate.

The role that the OECD and its program PISA aims to play in education, its content and its methodology can be summarize by their own self-assessment: "Over the past decade, the OECD Programme for International Student Assessment, PISA, has become the world's premier yardstick for evaluating the quality, equity and efficiency of school systems. By identifying the characteristics of high-performing education systems, PISA allows governments and educators to identify effective policies that they can then adapt to their local contexts." [7]

This self-evaluation takes place every three years, it is applied to students in the ninth grade, averaging fifteen years of age, and it targets three essential areas of the educational systems: Mathematics, Sciences, and Reading.

This peer-review process, after several phases of iteration, has identified those successful educational systems around the world that in turn serve as reference to the rest of nations. One sizeable example of this process is illustrated in the K-12 system in the United States where, during the Obama administration a set of criteria was stablished in the creation of the new school standards. The result of this process was the implementation for the first time in the history of the US of a standardized K-12 education system, called Common Core [8], and based on the educational systems of Hong Kong, South Korea and Singapore, the top nations in the PISA ranking.

This process of evaluating, selecting and replicating nominally successful educational systems affects the future of countries as well as the formative years of their students. Children around the world spend their most influential years in primary school. This period of education includes ten years of compulsory education that shapes their mind and their future, as well as that of their families, communities and countries.

Because of these ten years of education (5 to 15 years of age) are so critical and they affect populations from all

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countries of the world, a formal study and evaluation of the current educational systems and alternative spaces is pertinent, beginning by establishing a set of oppositional principles to carry these tasks. In this paper, three oppositional principles will be used: Education Content, Educational Methods, and Mind Development. These principles will address the questions:

- Is the Educational Content appropriate?
- Are the Educational Methods appropriate?
- Do the Educational Systems Allow an Optimal Development of the Mind?

In the following sections, we address these questions and present alternative responses of different constituent groups, including the de-facto Official answers (provided by the OECD) as well as from some alternative sources. On one hand we will find that the current world wide education systems (we will limit our analysis to mathematics) provides students with 10 years of unnecessarily fruitless experience as measured by the achievement tests and the content of the tests. At the same time, there are emerging fields that propose substantial alternatives.

# II. ANALYSIS OF THE CURRENT EDUCATIONAL SYSTEM

One invaluable consequence of the global adoption of the PISA standards is that it provides a comprehensive description of the materials evaluated, the principles used in their selection, and the achievement results obtained by the students after ten academic years of education in the systems. This extensive set of data will provide the background to address the three analytical questions that will inform the evaluation of the system:

- What content do nations of the world consider essential in education, and therefore is included in the curriculum?
- What type of tasks (Type-A or Type-B) are the students being trained to perform (these types will be described later)?
- What levels of accomplishment are obtained by the students after 10 years of education?

These questions include: What do we teach? What part of the brain do we develop? What do the children learn after ten years?

The developments of the last decades in the areas of cognitive sciences, evolutionary psychology and science of computation indicate that these three questions are interrelated: what we teach, and how we teach, determine the levels of accomplishment obtained.

### A. Content of Current Educational Systems

The first question is what do we teach in the classroom? This is probably one of the most unexamined questions in this field. One probable answer is 'we teach that which is important and necessary' both because is essential for the development of the educated mind, and because is essential for the constructive participation in society. However, it is quite possible that the current content of education does not properly address either one of these fundamental requirements. The content of the current educational system is based on the four areas of ancient mathematics: numbers, arithmetic, algebra, and Euclidean geometry (and in some cases basic Cartesian geometry).

Often these contents are justified because they are the legacy of the past, because it has been shown that students have difficulty learning these basic subjects and therefore other more advanced topics would be beyond their reach, and finally, because society requires citizens equipped with these skills in order to prosper. These and other historical reasons have created an obsolete curriculum.

### B. What Part of the Brain do Students Develop

The second question addressed in this section is 'what part of the brain do we develop?' Decades ago, when the current educational systems were designed, the model of the human mind as a blank slate suggested the role of the brain was to master content and methodology. It seems now absurd to overlook the computational capabilities and limitations of a computational entity such as the brain in order to accomplish computational tasks. Much has been learned in the last decades in the field of cognition in the human mind; however, no significant impact on the educational systems has yet taken place.

For this reason, the current educational systems, as indicated by every single problem in the PISA test, still base their mathematics education on Type-A problems. These problems have the following characteristics:

- They have a known and unique answer;
- They require a three step process for its resolution (Correspondence, Rule Identification, Numerical substitution CRN process);
- They require for their resolution the active involvement of System-2 [9].

System-2 is a serial processor, is slow, tires easily, and interferes, that is to say that it does not allow other processes to occur simultaneously.

In addition, type-A/System-2 problems use only descriptive mathematical languages. No single problem of the PISA Test uses generative mathematical languages, what we call Type-B problems. These characteristics, of course, have direct effects in the performance of students.

# C. What do Students Achieve

The third and final question is 'what do our students achieve at the end of ten years of education?' It would be expected that such long period of education would produce significant results.

The PISA results are of great significance, and during the last decade, they have gained prominence in the media, appearing in discussions of political and educational leaders. Unfortunately, these discussions involve world rankings and numerical averages that are generally disconnected from their cognitive significance. For example, the US obtained a PISA average of 470 in Mathematics in 2015 (a decline from 472 in 2012) while Norway, with its world recognized education system, obtained a PISA average of 502 (an increase from 501 in 2012.) From these averages, some conclusions can be obtained: Norway is slighting improving, while the US is slighting falling behind, and Norway maintains a significant advantage over the US. However, if the PISA averages 470 and 502 are put in context their significance changes.

PISA goes further, and quantifies the numerical results of the tests in terms of years of schooling in many of its reports. For example, in its summary report PISA 2012 Results in Focus, it includes 'Shanghai-China has the highest scores in mathematics, with a mean score of 613 points – 119 points, or the equivalent of nearly three years of schooling, above the OECD average.' [10]

Fig. 1 shows the latest world ranking in Mathematics according to PISA. This ranking is used to compare the performance of educational systems around the world.



Figure 1. World mathematics ranking according to PISA in 2016

Let us illustrate with some examples how these PISA averages in Mathematics are obtained, and the cognitive complexity they represent.

The first question of the PISA exam in Mathematics asks 'The attic of a farmer has a square shape. If each side of the attic has a length of 12 meters, what is the area of the attic? ' The correct response is 'the area of a square is side multiplied by side,' or 12 times 12, in this case, therefore the area is 144 square meters.

This problem had a difficulty level of 492 points in the PISA scale because only 61% of PISA students responded correctly to the question.

The second question of the PISA exam in Mathematics asks 'The sum of two opposing sides of a dice is always 7. Determine the number of dots on the opposing sides of these dice?' Fig. 2 shows the segment of the PISA test in Mathematics describing the problem of the dice and the sum of opposing sides.

In this photograph you see six dice, labeled (a) to (f). For all dice there is a rule: The total number of dots on two opposite faces of each die is always seven.



Write in each box the number of dots on the **bottom** face of the dice corresponding to the photograph.

(a)	(b)	(C)
(d)	(e)	(f)

Figure 2. PISA test in mathematics: Problem on the Sum of opposing sides of dice

This problem had a difficulty level of 516 points in the PISA scale because only 58% of PISA students responded correctly to the question. The test review documentation includes 'Answering this question correctly corresponds to a difficulty of 516 score points on the PISA mathematics scale. Across OECD countries, 58% of students answered correctly. To answer the question correctly students have to draw on skills from the reproduction competency cluster.'[11]

In this context, where the PISA index average of 500 is matched with a mathematical problem with the complexity of the problems described earlier, represents the complexity of problems of one single object and one single function. However, the current educational system expects only 50% of the students to reach the cognitive skills to solve correctly this type of problems.

The problems included in the PISA tests have been elaborated with detail and they target the various aspects of the curriculum that are assumed relevant in the education of students. The incorrect assessment is not caused by the lack of design the PISA tests. Rather, the reliance on obsolete principles of cognitive tasks and cognitive content is what makes these assessments fruitless and misguided.

We can summarize this section with these three answers, provided by the analysis of the content and statistics of the PISA tests, which indicate-that:

- Little knowledge acquired during the last decades in the area of human mind has entered into the K-12 Educational systems;
- No content of modern mathematics has entered the curriculum;
- The performance of the students is below levels of computational tasks of one single object (square, dice) with simple structure (one property and several procedures).

# III. EXPLORING ALTERNATIVE SPACES OF MATHEMATICS

The guidelines in exploring new spaces of mathematics that will replace the current model that could be described as the PISA model will include several pragmatic and research based questions. What new mathematical skills does the society of today need from its students and future active members in order to solve the challenging new problems that society faces? What methodologies that take advantage of the cognitive capabilities of the human mind and minimize its cognitive limitations will be used to educate our students in those new skills? What standards should be set given the needs of society and the capabilities of the mind? (Are only 50% of students capable of calculating the area of a square? or should 100% of students acquire the knowledge to for example create a self-driving car, an autonomous bouncing ball, or a three dimensional model of a small city and its building, parks and roads?)

# A. Introducing a New Curriculum

In the area of mathematic content, we propose the introduction in the curriculum of schools of topics in a space of mathematics that are characterized by three parameters:

- They use the massively parallel, language and object-based primitives of the human mind. (These resources become automatic, do not tire easily and do not interfere with other tasks.)
- They are based on generative languages that not only describe properties of objects, but also implement processes, change the state of other objects in the world.
- They address modern topics in Mathematics and in the modern world such as Discrete Calculus, Cybernetics, Probabilistic Thinking, and Differential Vector Geometry.

In the following sections, we will describe the structure and operation of one such project, a self-driving car.

# B. Developing a Different Part of the Brain

Much effort and study has been dedicated during the last two decades to understand the workings of the mind when involved in mathematical tasks, and much has been learned that explains its jarring limitations, and unimagined and unexplored capabilities [12]-[14] When college students of prestigious universities are confronted with the following problem (A bat and a ball cost \$1.10, and the bat costs \$1 dollar more that the ball. How much does the ball cost?), the vast majority of students incorrectly solves the problem, and at the same time they are unaware of their error [15]. These types of findings should not discourage the education profession, or justify low expectations on the cognitive abilities of the human mind. Rather, they should alert us of the limitations of an untrained mind, and at the same time establish a clear philosophy in education where we develop the uncharted cognitive resources of the mind.

Based on these seemingly paradoxical characteristics of the mind, the new spaces in mathematics should harness the generally untapped cognitive resources that the evolutionary human mind possesses. These guides include the use of parts of the brain with the following characteristics:

- They are able to represent and explore these complex systems (for example an autonomous butterfly and its behaviors, a self-driving car and its behaviors.) This system contains an extensive group of brain networks that represent and compute data in an object-oriented paradigm.
- They are able to represent and compute data using generative languages.
- They are able to learn through repetition, in particular with immediate and reliable feedback.

This paradigm of Object-oriented computation and data representation, of computation via a generative language, of learning via repetition and immediate and reliable feedback is what is missing in the current educational system.

Chess-masters, experience surgeons, pilots, piano players, group leaders, or teachers have acquired their expertise using this paradigm. Expert knowledge is automatic knowledge.

# C. Setting High Standards in Portfolio-Base Education

Standards based on test performance by definition sustain educational systems where the ability of the student cannot be measured otherwise [16]. They also sustain the idea that previous performance does not guarantee present success, thus the need of an independent sample of the student ability in an environment, the test, which usually requires the use of paper and pencil, and in general an experience of temporary stress, where the task to solve is short in time and numerical in nature.

The nature of these tests, short in duration and numerical in nature, match perfectly with the current system of mathematical education: Knowledge is presented in terms of short problems (PISA-test, type-A problems), with a specific known answer, and problems that can be solved with the use of paper and pencil, in general resulting in a numeric answer.

The type of tasks proposed for the new spaces of mathematics are generative in nature; they produce systems, as opposed to the descriptive numeric answers of traditional tests. These tasks are similar to the portfolio of an architect: the timeframe for their implementation is long and they are cumulative in nature. An architect with ten years of design and construction practice, or a student with ten years of active school design and construction create a portfolio of work that contains a comprehensive collection of the concepts acquired, experience gained, and work created.

The content, methodology and part of the brain used in the proposed spaces of mathematics are directed to the incremental and permanent construction by the student of a portfolio that emulates the portfolio of a surgeon, an artist, a pilot or an architect. Education is project based; activities are project based; and these projects constitute the building blocks of an education where the achievement of the students grows as the portfolio of projects grows in scope and complexity.

#### IV. ILLUSTRATING FUNDAMENTAL IDEAS AND COGNITIVE PROCESSES WITH A CASE STUDY

One of the areas that we propose we teach our children is the field of cybernetics, or self-regulated systems. For example we create a self-driving car; a car that is able to monitor its environment and find the optimal way of navigating through it. A car that is able to navigate through circuits never seen before.

We use this example to illustrate some of the fundamental ideas that these proposed new spaces of mathematics present to the students. In addition, this example will illustrate the cognitive processes developed in the students' mind. It will also illustrate how these processes differ from traditional school tasks.

#### A. Selection of the Reference Problem

One powerful strategy for solving complex problems is called 'wishful thinking.' The main concept is to proceed in the process of design as if all problems already have a solution. Assuming the problems have solutions, the student concentrates in the design process [17].

The examples that follow are used to illustrate this strategy and other fundamental principles. We will use the Scratch programming environment for the implementation of these projects.

In the case of the self-driving car, we begin by (1) placing the object of the car in the middle of the road, and see what happens. Indeed, the car does not move. Therefore, it is obvious that it needs some means of locomotion. To obtain that locomotion (2) we add an engine (move block). We observe that it moves, but only a small distance. (3) We need to maintain this motion, so we repeat the process indefinitely (forever block). Now the car keeps moving. However, there is a problem: the car keeps going where there is no road. (4) We address this problem by having the first controller on the car. This controller will look at the environment with a sensor. If the sensor detects the car is over the road, the engine will be on. If it detects that is getting off the road it will stop the car.

Fig. 3 shows the state of the self-driving car with the basic engine control: the car will move forward as long as the road detector guarantees there is road ahead.

This is already a great success. The car is able to drive and then stop when it is about to exit the road. The reason that the car was about to exit the road is that the car was going straight when it encountered a turn on the road. For that (5), we will provide another sensor that will detect a turn before the car exists the road, and will turn the steering wheel.



Figure 3. Initial steps in the implementation of the self-driving car: Continuous movement over the Road

It is important to note that the mechanisms for detecting turns are philosophically similar to the mechanism that controls the engine: Detect and event, and cause an appropriate action. With this new mechanism, when the car encounters a left turn, it causes the wheel to turn left.

In addition, here two important factors need to be considered. It really does not matter how much the steering wheel is turned. It will keep turning until it satisfies its requirements. This is a robust system. The power of the action is the idea itself, no the particular details. The details are so unimportant that they do not need to be implemented.

A second important factor is that the control of the steering wheel is completely independent of the control of the engine. There is no communication between them, and there is no overall control over these separate processes. These are parallel, independent, and unsupervised process.

# B. Principle of Emergence and Parallel Processes

This concept of parallel, independent, and unsupervised process underlies the principle of emergence: the process by which complex, intelligent systems arise from multiple unintelligent processes.

This is a fundamental idea in Computational thinking: there is design without a designer [18] there is intelligence without intelligent components.

In fact, the implementation of this type of control is the collaboration of two simpler and less intelligent units. The sensor of the road does not have to think in order to operate. It could be as simple as a detector of light. When it sees the color of the road is silent, when it sees another color it produces a signal.

Imagine a crow sitting in the front bumper of the car. It is usually bored by the color of the road. However, when the car is about to exit the road, it sees the green of the grass, and that makes him happy, and screams of happiness. Inside the car, sitting on the on/off switch of the engine, there is a turtle. The turtle likes to sleep over the switch of the engine, so the engine is always on. However, when it hears as scream, it jumps scared off the switch, and the car stops. When the screams go away, the turtle goes to sleep, and the car moves again. Neither the crow nor the turtle are interested in driving the car, nor do they know that they are collaborating. They are parallel, unintelligent processes that happen to cooperate. Nevertheless, the cooperation is unintended, and unsupervised. No superior entity analyzes these processes to take some action.

Currently the car has two controls. One will allow it to stop if it ever gets off the road; the other allows it to turn left if it encounters a left turn. With these two controls, the car is only able to travel in circuits with only left turns. Let us include now all possible circuits by adding a control that will detect right turns and will accordingly turn the steering wheel to the right. Now with these simple, parallel, independent, and unsupervised controls, the car can drive in any, never seen before, circuits. Fig. 4 shows the state of the self-driving car where in addition to the engine control, two other scripts have been added in order to detect turns on the road and activate the corresponding steering actions.



Figure 4. Incremental development: Adding detection of left and right turns and steering capabilities

#### C. Incremental Evolution of Complexity

At this point in the design process, the student is prepared to take the car to any degrees of incremental sophistication. For example, (6) by adding a long distance detector in the front of the car (an eagle that looks into the distance), it would be possible to detect when the car is in a long stretch of straight road, or when it is approaching a turn. And by having an engine that can run at different speeds, we can make the car go fast during long segments of straight road, and slow down during a turn, whether the turn be to the left or to the right, and later speed up again when it exists the turn.

In a few moments, we have experienced the transformation of a car that was sitting still on the road, into a car that is aware of its environment and it is able to travel any road in an efficient and safe mode.

#### D. Cognitive Processes in the Mind of Students

If we focus our attention on the processes in the mind of the student, we can learn some fundamental principles to be applied to the learning process. It is obvious to the observer that the student understands that what makes the car move, turn, speed up or slow down, is a set of scripts.

The difference between a still car and an intelligently moving car is a group of scripts. These scripts constitute the elements of a generative language. When a student solves a mathematical problem such as those of the PISA exam, it follows a process of three steps [19], and the result of this process is a numerical result that describes some property of the topic studied, for example the area or the perimeter of a square. This is the result of using a descriptive language. Descriptive languages can only find out something about a topic, they cannot create something new. They cannot create a car that started as a dead car and ended as a living car.

#### E. Cognitive Processes in Descriptive Languages

As a contrast to the creation of a self-driving car, let us analyze the cognitive processes that the mind can engage when solving a PISA-type traditional task. Imagine that you ask a student to multiply mentally 23 x 79.

Because we know that a computer can do this computation with no effort, and that the hardware required to implement this computation contains only a few logic gates, we know that the level of the cognitive processes required is simple.

That does not prevent the fact that we humans have great difficulty implementing these types of tasks, that they require great effort on our part, and that implementing hundreds of multiplications does not develop new cognitive skills in our mind.

Next, ask a computer the following question: 'The trophy would not fit in the brown suitcase because it was too big (small). What was too big (small)? Answer 0: the trophy. Answer 1: the suitcase.'

This is an example of the Winograd schema [20] Today no existing super-computer can successfully solve this type of problems. The reason why the best supercomputer in the world cannot solve this type of problems is that they require processing data at a cognitive high level. However, a young student has no problem finding the answer without any apparent effort (although with great internal computation), and the young student has no problem explaining the reasoning that was followed to arrive to the correct answer.

#### F. Cognitive Processes in Generative Languages

Ask the young student why the car was not able to move at the beginning, or how it is now able to slow down before a turn and you will hear an effortless and detailed explanation that no supercomputer can provide today. The explanations that the young student provides are the description of the ideas that his mind now has acquired and that are coded in terms of a generative language. It is not only that the young student knows now how to create a self-driving car, and that he can explain to others how to build it, and how it works. What is more important is that the students spent their time in school having their minds working at high levels of cognitive computation [21], [22] (that no current supercomputer can match). In addition, in this process, they are learning a new language, a generative language, and new ideas coded in this language.

#### G. Innate Capacity for Generative Languages

To illustrate this new power, let us ask a student "Imagine a small island, a palm tree, a coconut, man and

his monkey friend. Can you create a funny story that contains all these elements? Can you create a boring story, a sad story?

If we give our students a language and a few primitives in the areas of cybernetics, discrete calculus, probabilistic thinking, vector geometry, they will create fantastic complex worlds. We will provide for them a world where they spend most of their time in a high cognitive level world. They will build in their minds a language and primitives that will permanently expand their horizons. They will be able to teach others and explain with detail what they know, and even though their minds will be producing billions of computations, like when we invent a story, the experience will be effortless and pleasant. Fig. 5 shows the state of the self-driving card projects where the student has implemented several advanced features: Multi-track Testing, Multiple Cars, Multi-Speed Control and Sound Effects.



Figure 5. Advanced incremental development: Multi-track testing, multiple cars, multi-speed control, sound effects

Why we still do not do these things, and insist in doing PISA type-A problems, which are effortful, unpleasant and useless is a mystery that perhaps only the human mind can solve.

#### V. DISCUSSION AND CONCLUSIONS

In this paper we use a three phase approach in order to validate a set of alternative spaces of mathematics for K-12 education that are intended to optimize the schooling years of students around the world and to address the cognitive skills that today's society requires.

It is relevant to note that despite significant progress in cognitive sciences and computational based education in the last decades the present environment of mathematics education around the world has become a monolithic entity regulated by global standards, obsolete content and failed pedagogical strategies.

The three oppositional principles used in the analysis of existing and proposed programs address directly the fundamental questions 'What educational content is appropriate? What educational methods are appropriate? Do the educational systems allow an optimal cognitive development of the mind?'

In the analysis of the current global system of mathematics education, the content studied in the classroom is reduced to the four areas of ancient mathematics: numbers, arithmetic, algebra, and Euclidean geometry (and in some cases basic Cartesian geometry). The type of problems studied in this system is limited to Type-A problems: they have a known and unique answer; they require a three step process for its resolution (Correspondence, Rule Identification, Numerical substitution – CRN process); and they require for their resolution the active involvement of System-2.

Regarding the accomplishments of the students after ten years of schooling, according to the PISA test, the nations of the world hover around the OECD average of 500 points. However, detailed analysis of the meaning of this numerical reference shows that a simple problem, such as the calculation of the dots on a dice corresponds to a difficulty of 516 score points on the PISA mathematics scale because across OECD countries, only 58% of students answered correctly. It is difficult to justify ten years of mathematics education to achieve such mediocre goals.

The proposed new spaces in mathematics are characterized by the following properties: they use massively parallel languages and object-based primitives that are existing resources of the human mind; they are based on generative languages that describe properties of objects and implement processes; and address modern topics in Mathematics such as: discrete calculus, cybernetics, probabilistic thinking, differential vector geometry, and others.

These domains allow students the representation and exploration of complex systems, the representation and computation of data using generative languages, and the opportunity to learn through repetition with immediate and reliable feedback.

The achievement of the students in this paradigm is not measured by temporary success in ad-hoc tests, but rather in the compilation of career portfolios that document the skills mastered during the schooling years.

Finally, an example of a self-driving car project is described in order to illustrate some of the fundamental ideas behind the new proposed spaces in mathematics. These ideas include the principle of emergence, the power of parallel processes, the principle of incremental complexity, the analysis of cognitive processes in the mind of students, the analysis of cognitive processes in descriptive languages vs. generative languages, and the innate human capacity for the use of generative languages.

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