

An Investigation of Students' Difficulties in Solving Non-Routine Word Problem at Lower Secondary

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Abstract—This study explores the cognitive processes undergone by Mauritian students who have difficulties in solving word problems. A questionnaire of three non routine word problems was administered to 190 grade 8 students, of different abilities. 15 students were interviewed to gauge the cognitive processes used while solving the problems. Montague's (2003) framework for problem solving was used to analyse the data. All students could read the problems. High Achievers (HA) are wrong due to careless errors. While HA demonstrate good problem solving skills, some exhibit overconfidence. Average Achievers suffer from procedural errors while Low Achievers face difficulties mainly in visualizing and representing the problem.

Index Term—problem solving, lower secondary, cognitive functioning

I. INTRODUCTION

Word problems have for long been a major part of school mathematics for many reasons. For instance, they provide opportunities for students to use mathematical tools, promote the link between mathematics and real life context, use thinking and problem solving heuristics, and provide a platform to develop new concepts and skills [1]. However, several critics [2] have questioned such justifications. [3] has even highlighted that word problems are often mere disguise for practice of the four basic operations.

Different types of word problems have been the focus of much analytical and empirical research in the past, for instance, in the classification of addition and subtraction word problems [4] and in multiplication/division situations [5]. Multi-step word problem is another type which involves two or more steps in solving process. Studies reported that solving mathematical word problem with two or more steps create more difficulty in obtaining the right answers [6].

The sources of difficulties with word problems are well documented. For instance, many studies observed ineffective instruction as one of these sources [7], while others suggested a lack of linguistic knowledge [8]. The conditions are more challenging when the word problems are in the learners' second language [9] among others.

While in the 1970s, a lot of attention was devoted to the study of determinants of word problem difficulty [10], the 1980's witnessed the emergent of frameworks that described the problem solving process and suggested reasons behind successes or failures [11]. The 1990's saw the evolution of models of teaching using problem solving [12]. There is general agreement today that problem difficulty is not so much a function of various tasks variables as it is the characteristics of the problem solver [13], [14]. Recent researches focused on the thinking processes used by individuals or small groups as they solve problem or as they reflect back on their problem-solving efforts. [13] suggested that more attention should be given to groups and whole classes.

A. Cognitive Process

Cognition is the study of how the mind works [15]. Cognitive processes are the mental processes of an individual. Many theories based on cognitive development or processes were developed; the most cited being those of Piaget and Vygostky. Research evidences suggested that the mind has internal mental states such as beliefs, desires and intentions. Cognitive processes may be understood in terms of information processing, especially when a lot of abstraction or processes are involved.

Several frameworks of problem solving have been created. [16] four phase (understanding, planning, carrying out the plan and looking back) description of problem solving procedures is a prototype on which recent mathematical problem solving research has been based. [11] argued that four factors (knowledge base; problem solving strategies - heuristics; Control - monitoring and self-regulation; belief) are necessary and sufficient for understanding the quality and success of the problem solving attempts. Many other similar frameworks were generated [17]-[19]. In this study, the framework proposed by [19] will be used to explore the cognitive processes associated with problem solving and which she established as follows:

Reading: To understand each part of the problem to establish relationships among the parts.

Paraphrasing: To translate the linguistic information in the problem by rephrasing or restating the problem.

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Visualizing: to process the linguistic and numerical information in a mathematical problem and form internal representations in memory.

Hypothesizing: to develop a solution plan that is linked to the comprehension of the problem and the integration of the problem information. Reading and representation strategies assist problem solvers in deciding on a solution path.

Estimating: to accurately predict an outcome based on the question and the goal that was set.

Computing: to perform the correct operations and also recall the necessary mathematics fact for accuracy.

Checking: to verify both the process and the product.

B. Research Evidences on Word Problem Solving

Research evidence showed that for mathematical problems solving, the most important deficiency seems to be in problem representation processes and strategies, which are critical in effective problem solving [18]. [20] stressed the importance of problem representation strategies to process linguistic and numerical information, comprehend and integrate the information from internal representations in memory and develop solution plans. According to [21] and [18], problem representation strategies facilitate translating and transforming problem information into problem structures or descriptions that are verbal graphics, symbolic and/ or quantitative in nature. These verbal and visual representations in turn assist in organizing and integrating problem information as the problem solver develops a logical solution plan.

[22] examined the verbalizations of middle school students as they thought aloud while solving three mathematical word problems. They observed that students tend to stop verbalizing or verbalize incompletely when under high cognitive load. Gifted students were found to be more strategic in their approach to problem solving than less proficient problem solver. Learning Disability (LD) students lack knowledge of problem representation strategies and rely more on reading, computing strategies than other students. [22] observed that students with LD lack knowledge of critical strategies which may lead to incorrect solutions.

[23] found no difference on reading achievement, cognitive ability and computational accuracy measures between students with LD and average achievers. They suggested that their poor performance was due to other factors such as inability to judge the difficulty level of the problems, poor perceptions of their ability to solve problems, lack of persistence and strategy deficits. [23] also suggested that students do not have the cognitive tools to perform the task and consequently spend their time using ineffective trial and error strategies. [23] found that gifted students have a repertoire of problem solving strategies that are easily and quickly accessed as task demands increase.

The literature on problem solving shows that the topic is well documented. However, most of the research conducted has focused on small group or individuals. There is a lack of research conducted on intact class as

highlighted by [13]. Moreover, the literature suggests that problem solving is more a concern of the characteristic of the problem solver rather than problem difficulties. Consequently, the present study investigates the cognitive processes undergone by students working under classroom condition. Interviews were conducted to explore the difficulties encountered by students of different abilities (high, average, low) with respect to [19] cognitive processes among others. The study is the first in its kind in Mauritius.

II. METHODOLOGY

A. Sampling

Three questions were administered to 190 grade 8 (13-14 years old) students from six intact classes. One class (boy) was from a high ability school, two classes (boy) were from an average ability school and the remaining classes (two boys and one girl) were from three low ability schools. In Mauritius, students are admitted to secondary school based on the national grade 6 examination result. There are some high demand (star) schools (high ability) and other schools which grouped students of comparable abilities, that is, either low or average. Based on their work, 15 students were interviewed to explore the cognitive processes through which they have gone during problem solving. The students were interviewed either shortly after they solved the problems or on the following day. Students were given a form (Framework Assessment Form) to fill just after they completed the problem to gauge the different cognitive stages they gone through while working.

B. Framework Assessment Form

A set of questions based on [19] problem solving framework was developed and organized according to the following cognitive processes: read, paraphrase, visualize, hypothesize, estimate, compute and check. In all, 17 short questions were designed to have an account of the cognitive processes undergone by the students while solving the word problem. Students were expected to tick the different processes they have gone through.

C. Interviews

The scripts were scrutinized and were chosen based on the difficulties the student(s) had in solving the problem. Few scripts with correct workings were also selected for comparison purposes. The corresponding students were then interviewed so as to shed light on the difficulties and cognitive processes they undergone while solving the problem. Semi-structured interview was used and oriented towards the cognitive processes proposed by [19]. The main questions were:

1. Read the question aloud.
2. Explain what you have understood.
3. Explain what you have done.
4. Before you completed the work did you have an idea about the answer?
5. Did you check your work?

D. Analysis of Data

Audio recorded responses from interviews were transcribed verbatim and the transcripts were read several times to extract relevant facts in line with research objectives and for other emerging themes. Interview data were then analysed using mainly [19] cognitive framework of problem solving. The cognitive processes used by students in solving word problems were triangulated from different perspectives, that is, data collected from student's script, Framework Assessment Form and interviews.

E. Ethical Issues and Limitation

Ethical considerations were observed in the research process to protect the rights of the respondents. Permission was granted by the Ministry of Education and Human Resources to access the schools and to collect data for the survey. The purpose of the study and the instruments for gathering data were explained in the letter addressed to the Ministry. The researcher then approached the prospective rectors, deputy rectors and the teachers involved and gave details of the research. Arrangements were then made for the running of the study. Subjects were also briefed on the purpose of the study and were ensured confidentiality of the data gathered.

A limitation is that the interview technique used to analyse the student's difficulties while solving problems might have some shortcomings as some students might not be able to construct an exact account of their behaviours. Also, the difficulties faced in solving problem are not in any way an exhaustive list owing to the limited number of questions used.

F. Word Problems

The problems were adapted from Cambridge local examination papers and Australian Mathematics Competition questions. The items are as follows;

1. In a queue at the bus stop, Sam was behind Sarah and Peter was between Sam and Sarah. Jack was behind Sam and Jane was in front of Sarah. Arrange them in order and state who was fourth in the queue?

2. A single DVD costs Rs 9. A special offer pack of 10 DVD costs Rs 78. Jane bought a special offer pack instead of 10 single DVD, calculate her saving.

3. A carpenter made chairs and stools. Altogether, he made a **total** of 14 items (chairs and stools) with a total number of **51 legs**. If the chairs each have **4 legs** and the stools each have **3 legs**, how many chairs did he make?

G. Sample of Interview

An extract of S11's work is show in Fig. 1. In the interview sample that follows, R stands for the researcher and S11 for the student with identification number 11.

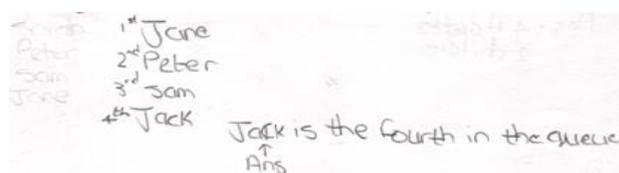


Figure. 1. Extract of student S11 working for Item 1.

R: Read the question aloud

S11: (Reading the question with some difficulty)

R: Explain what you understood

S11: Sam was behind Sarah and Peter, Peter is between Sam and Sarah

Jack is behind Sam and Jane, Sarah was in front of Jane

R: Can you repeat this part. "Jack was behind... Sarah"

S11: hm . . . , Jack is behind Sam, Jane is in front of Sarah

R: Explain what you have done

S11: I placed Jane first

Sam, Sarah and Peter

I placed Peter between Sam and Sarah

Then I placed Jack behind Sam and Jane

Jane (1st), Peter(2nd), Sam (3rd) and 4th

Jack

R: Before you completed the work did you have an idea about the answer?

S11: No

R: did you check that you have correctly completed the problem?

S11: Yes

R: Can you explain how you checked it?

S11: Checked only the 4th one.

S11 read the problem with some difficulty and could hardly paraphrase it, translating word by word and giving the impression that he is focusing on the bits and parts of the problem. He could not develop a clear picture (visualize) of the whole problem. This may explain why when he erased the solution to rewrite it, he unknowingly missed Sarah. According to the Gestalt Theory, the essence of successful problem solving behavior is being able to see the overall structure of the problem [24] and this is what S11 seems to be lacking. During interview, S11 showed lack of confidence and he could not see the connection between the elements. He checked (poorly) his work by just looking at the fourth person in the queue. He did not reread the problem nor recheck the ordering of the names. This denied S11 the opportunity to clarify the problem and detect any error. Despite having used six of the seven cognitive processes outlined by [19], S11 could not obtain the correct answer as he poorly paraphrased and wrongly visualized the problem, and did not reflect seriously on his work.

III. MATH

It was found that Low achievers face difficulties in decoding the language in the problem (as seen by [1]) and could hardly visualize the problem. Carelessness either in reading or in calculation was found to lead to wrong answer, as observed by [25]. It was found that miscomprehension (as observed by [26]) and poor procedural skill (as noted by [21]) lead to wrong answer. Once a student was unable to visualize or hypothesize the problem, it was almost impossible to obtain the correct answer. High achievers were found to be good problem solvers (as noted by [11]). Student's confidence may

influence success in solving word problem (as noted by [27]). An important observation made is that the first impression made by students about the problem is very dominant and difficult to change. The cognitive processes resulting from analysis of interviews using [19] framework

are summarized in Table I. The following notation are used:- \checkmark : Perform process correctly; O : Perform process poorly; X: Did not perform the process; H: High; A: Average; L: Low.

TABLE I: SUMMARY OF COGNITIVE PROCESSES USED STUDENTS FOR THE THREE ITEMS

| Student/Item | Ability | Read | Paraphrase | Visualize | Hypothesize | Estimate | Compute | Check | Answer |
|--------------|---------|--------------|--------------|--------------|--------------|----------|--------------|--------------|---------|
| S1/1 | A | \checkmark | \checkmark | \checkmark | \checkmark | X | X | O | wrong |
| S1/2 | A | \checkmark | \checkmark | \checkmark | \checkmark | X | \checkmark | O | correct |
| S2/2 | A | \checkmark | \checkmark | \checkmark | \checkmark | X | O | X | wrong |
| S3/3 | A | \checkmark | \checkmark | O | O | X | \checkmark | X | wrong |
| S4/3 | A | \checkmark | \checkmark | \checkmark | \checkmark | X | \checkmark | X | wrong |
| S5/3 | A | \checkmark | \checkmark | O | O | X | \checkmark | O | wrong |
| S6/1 | H | \checkmark | \checkmark | \checkmark | \checkmark | X | O | \checkmark | wrong |
| S7/2 | H | \checkmark | \checkmark | \checkmark | \checkmark | X | \checkmark | \checkmark | correct |
| S8/3 | H | \checkmark | \checkmark | \checkmark | \checkmark | X | \checkmark | \checkmark | correct |
| S9/3 | H | \checkmark | \checkmark | O | O | X | \checkmark | X | wrong |
| S10/1 | L | \checkmark | \checkmark | O | \checkmark | X | \checkmark | X | wrong |
| S11/1 | L | \checkmark | O | O | \checkmark | X | \checkmark | O | wrong |
| S12/1 | L | \checkmark | \checkmark | \checkmark | \checkmark | X | \checkmark | \checkmark | correct |
| S13/2 | L | \checkmark | \checkmark | \checkmark | \checkmark | X | O | O | wrong |
| S14/2 | L | \checkmark | \checkmark | O | O | X | \checkmark | O | wrong |
| S15/3 | L | \checkmark | O | O | \checkmark | X | \checkmark | X | wrong |

Note: \checkmark : Perform process correctly; O : Perform process poorly; X : Did not perform the process; H: High; A: Average; L: Low.

IV. CONCLUSION

Based on [19] cognitive processes, the following observations were made. Almost all students could read the problem and understand what was asked, however difficulties arose mainly due to inability to develop a correct structure of the problem. High Achievers (HA) were wrong in solving word problems mainly due to computation or careless errors. While some HA demonstrated good problem solving skills by verifying their work, some showed overconfidence and did not check their solution seriously. Average Achievers (AA) were wrong mainly due to computation error and inability to visualize the problem correctly. Low Achievers (LA) were wrong mainly due to their inability to visualize the problem correctly and consequently could not develop a correct plan to solve the problem. Some LA showed difficulty in paraphrasing the problem and/or poor processing skill in their solution. In general, most students did not check their solution properly or showed a lack of strategies to check their problem. The findings suggest: that the teaching process must lay emphasis on reading and interpretation of problem; students, of low abilities in particular, should be encouraged to use various representations and strategies to solve problem; students must be urged to develop strategies to check their answers.

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