Solving Complex Problems with a Computational Mind: An Alternative to Heuristic Search

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Abstract **— A critical set of advances in the world of cognitive sciences during the last two decades is redefining the directions of research of cognitive processes. A set of obsolete cognitive principles has been identified and new methods and objectives have been set for the study of complex problems. We investigate in this paper one such problem in an experiment with students (n=192) from four universities in Spain. The experiment reveals that (i) students easily and reliably acquire (appropriately designed) complex algorithms and (ii) students learn and apply these algorithms in an affective state of ease. These students systematically outperformed published results in an isomorphic task (inheritance genetics). These results indicate that appropriate data encoding, explicit algorithmic definition, and the activation of human cognitive primitives is sufficient to accomplish the task.**

Index Terms**—heuristic search, computational thinking, object oriented thinking, information encoding, algorithmic thinking**

I. Introduction

Heuristic search is the gold standard of problem solving in the current educational systems in the areas of science and mathematics. During the last few decades there had been numerous advances [1]-[9] in the areas of cognitive science, neuroscience and computation that suggest that both the content and teaching methodology used in K-12 education should be redesigned. In this paper, the results of an experiment are described, showing how some of new these ideas could be integrated into the classroom and what their impact might be.

A. The PISA Exam and Type-A Problems

The Programme for International Student Assessment (PISA) [10], [11] is an international test which is intended to evaluate the quality of education systems around the world. Ninth-grade students (15 years-old) of OECD member and associated countries are tested every three years in areas such as mathematics, science, and reading. The test results of over half a million students representing more than 70 economies are collected in a series of comprehensive reports that describe the current state of their education systems, and how they compare with all the other participating countries. Because of the worldwide scope of the test, and the country-based rankings derived from the tests, the PISA tests are acquiring an increasingly important role in governmental programs, media reports and assessments, and in the general culture of society.

In particular, the PISA test of mathematics consists of fifty problems [12]. All the problems in the test belong to the group of Type-A problems [13]. Concisely, a Type-A problem is one that requires a three-step process for its resolution: correspondence, rule identification, and rule application.

One of the most widely used intelligence test, and the most difficult non-verbal intelligence test, is the Raven test [14], [15]. It consists of 32 Type-A questions, each increasingly more difficult than the one before. The questions increase in difficulty because each of the three phases of the problem were designed to include greater difficulty. It is therefore expected that students performing well in the Raven intelligence test will succeed in areas such as mathematics or sciences since they draw on the same cognitive skills: heuristic search and application of rules.

The first question on a recent PISA mathematics test [12] described a room with a square shape, with all sides of length 12, and it asked to determine the area of the room. The phase of correspondence consisted in determining that: the problem dealt with a square, the side was known, and equal to 12, and that the area was asked to be calculated. The phase of rule identification consisted in recognizing that square, side, and area were related by an equation: Area (of a square) equals side times side. The last phase, rule application, required simple arithmetic and no algebra: 12x12=144.

What is more significant in this example is that this problem corresponds to a level of difficulty of 492 score points in the PISA mathematics scale. Across OECD countries, 61% of the students answered correctly. In other words, 4 out of 10 fifteen-year old students around the world were unable to solve this problem correctly.

B. Proliferation of Type-A Problems

The paradigm of Type-A problems is extremely attractive for an education system. A single three-phase

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procedure can be applied to an ever increasing library of problems in diverse fields: geometry, trigonometry, physics, chemistry, biology, etc.

Any system where known and unknown data can be represented with numbers, and where their relationship can be represented by equations, is ideally suited for this paradigm.

In an education system where the fundamental means of problem resolution were memorization, and calculations using pencil and paper, the paradigm of Type-A problems was stablished with solid foundations.

C. One Mind – Two Systems

The mind operates in two distinct and very different modes [16]: consciously applying a set of rules (System-2), for example multiplying 27x14; or automatically, effortlessly, and unavoidably (System-1), for example recognizing a friendly face.

In general, the problem solving techniques used by the students while solving a Type-A problem involve a process of heuristic search. They confront the problem in the same way a novice chess player attacks a novel board distribution: they read the problem for clues that will guide them in the process of finding the optimal strategy.

This search, like any conscious search, is implemented by System-2. The student needs to set a goal, actively work towards achieving the goal, while continuously monitoring this mental activity.

System-1 is automatic and relies on knowledge already stored in long-term, permanent working-memory. System-1 implements these tasks automatically and effortlessly. Expert knowledge is automatic knowledge.

System-2, however, is the only one capable of implementing self-directed, heuristic search. Unfortunately, these tasks are effortful, and System-2 tires easily.

D. Towards a System-1-Based Education

The current systems of education value and promote strategies of problem solving based on heuristic search. There are good reasons why this paradigm became the accepted status-quo: the ideal concept of humans as reasoning creatures, as opposed to automatic, intuitive actioners, and the pervasive idea that a Raven-type intelligent test is a required and accurate discriminating tool of character among human beings.

In the process of developing System-2-based education systems, which would help identify and separate the different levels of cognitive accomplishments among students, a vast area of System-1 education was, and in many cases still is, neglected or abandoned.

This critical mistake is not shared by all training or education systems. In areas such as commercial aviation, medicine, the military, etc., students are educated with a method that guarantees that knowledge is permanently stored in long-term working memory in such a way that it is automatically accessed, and effortlessly. Expert knowledge is automatic knowledge.

E. Learning from the Grandmasters

A critical finding in cognitive sciences is that expert knowledge is automatic knowledge that is domain-specific and stored in long-term working memory. Groot and others [17]-[19] studied the mental processes of experts in several areas of knowledge. It was assumed, and it is still a generalized assumption, that chess grandmasters are among an elite minority of human geniuses with supernatural cognitive capabilities. This assumption is incorrect. Grandmasters are made; they are not born with particular cognitive gifts.

In particular, the general assumption in the population is that chess grandmasters, when analyzing a move, have the ability to explore many more options than a novice player, and also that they are able to explore many more future moves. This greater ability to explore in breadth and depth is what, it is assumed, gives them their remarkable ability. If this were the case, the implication would be that their System-2 had outstanding properties of search. But in fact, the number of exploratory moves does not significantly differ between novice and expert players. Grandmasters don't even have greater abilities to memorize random distributions of pieces on a chess board.

The difference is that grandmasters automatically recognize an existing distribution of pieces. They also retrieve form long-term working memory the most advantageous moves for that distribution. It is equivalent to dropping a blindfolded expert traveler in the middle of a city, and when the traveler removes the blindfold, she recognizes the city and the street where she is, and suddenly all the relevant information about the city becomes effortlessly available. A novice traveler in the same situation, never having been to that city before, has no long-term memories to access, and it will have to rely on maps an effortful search.

F. The Computational Limitations of Rational Thinking

Discoveries in cognitive sciences have identified the general methods by which humans try to acquire knowledge and solve problems.

In the 1970's Daniel Kahneman [20], [21] performed breakthrough research in the area of decision making, which later was recognized with the Nobel Prize in economics. A key finding of his work was the counter-intuitive idea that when we make decisions we use simple heuristics that answer a simpler question that the one we are trying to solve. Then, we claim to have solved the difficulty problem. A critical implication of this finding is the intrinsic computational limitation of System-2 when solving problems that require reasoning.

What is perhaps more relevant about this finding is that, four decades later, the general assumption still is that System-2 consistently solves problems through reasoning. In this view, a strong, reliable System-2 is the panacea of human reasoning, and therefore justifies an educational system heavily dependent on System-2.

Unfortunately for this obsolete theory of the mind, the evolutionarily process of the human species has assigned System-2 a different role, and its capabilities are limited.

II. Inheritance Genetics Problem

A goal of the experiment presented in this paper was to compare the traditional heuristic search method normally

used in our education systems with an alternative computational method specially designed to take advantage of the cognitive primitives of the human mind. The first step was to select a published problem that would serve as reference.

A. Selection of the Reference Problem

From the published literature, a target study was selected with the following characteristics: the difficulty of the problem had to be significant, therefore allowing only a limited segment of the student population to be able to solve it; and the subject topic was general enough so that many readers would be able to identify with the experience of fellow college students. A study [22] in an introductory college course in genetics was selected, and among the problems discussed in the article, the questions labeled as difficult were selected.

In the published study, after the professor lectured about the topic, students correctly responded to a multiple-choice test at a rate of 20%. Later, after students were encouraged to discuss among themselves the different aspects of the problem, the rate of correct responses increased to 50%.

The selected problems, as well as all the problems listed in the study, were Type-A problems. Type-A problems are suitable for multiple-choice tests since the answer is known before the students attempt solving the problem. The second characteristic of Type-A problems is that they can be solved with a simple, linear algorithm.

B. Algorithmic Implementation: A Java Program

Following the published guidelines [18] to formally describing knowledge using computational means, we created a java program (Table I) in order to solve the selected genetics problem (x-linked vs autosomal inheritance). There were two main goals for the creation of the java program. The first was to quantify the data structures that were involved in the problem: what was known, what was unknown, what was relevant, and how much short-term memory processing was required from the student in order to comprehend the problem. The second goal was to specify the concrete set of rules, and their order of application, in the resolution of the problem.

The java code listed in Table I shows that, to represent the data of each problem, a total of six arrays are required. The size of the arrays may vary from problem to problem without significantly altering the complexity of the problem. The data included in the six arrays is presented in the Biology course in two separate but related diagrams: a family pedigree diagram, which includes the members of the family and their phenotype information; and a microsatellite diagram with the genotype information.

TABLE I. ALGORITHMIC IMPLEMENTATION OF THE GENETICS INHERITANCE PROBLEM

```
/**
 * Algorithmic implementation of the Genetics Inheritance Problem
 *
 */

import java.text.*;

public class Mode_Genetics_Inheritance {

 public static void main(String[] args) {

  // Declare and initialize variables-------------------------------------
  String names[] = {  "I-1","I-2","II-1","II-2","II-3","II-4"  };
  String sex_M_F[]    = {  "F"  ,"M"  ,"M"  ,"F"  ,"M"  ,"F"    };
  String age_A_C[]    = {  "A"  ,"A"  ,"C"  ,"C"  ,"C"  ,"C"    };
  String affect_Y_N[]  = {  "N"  ,"N"  ,"N"  ,"N"  ,"Y"  ,"N"    };
  String code_Left[]   = {  "A"  ,"B"  ,"D"  ,"A"  ,"A"  ,"C"    };
  String code_Right[]  = {  "D"  ," "  ," "  ,"B"  ," "  ,"D"    };

  String affect_Symbol = "Not Known yet";

  // Determine if X-linked-----------------------------------------------
  boolean xlinked = true;

  for (int i=0; i<sex_M_F.length ; i++){
            if(sex_M_F[i] == "M"){
                        if(code_Left[i] != " " && code_Right[i] != " "){
                          xlinked = false;
                        }
            }
  }

  // Case X-linked find male affected-------------------------------------
  boolean male_affected = false;
  int index_aff = 0;

  if(xlinked == true){

    while( male_affected == false){
```

```
            if(sex_M_F[index_aff]  ==  "M"  &&  affect_Y_N[index_aff]
== "Y"){
                male_affected = true;
               }
         index_aff++;
  }
   index_aff--;
}

// Case X-linked find female with same card color-------------------------
boolean female_card = false;
int index_fem = 0;

if(xlinked == true){

     while( female_card == false && index_fem < code_Left.length){
            if(sex_M_F[index_fem] == "F" &&
                        (          code_Left[index_fem]                    ==
code_Left[index_aff] ||
            code_Right[index_fem]== code_Left[index_aff]   )  ){

                 female_card = true;
               }
            index_fem++;
     }
  index_fem--;
}

// Case X-linked find if female is affected ------------------------------
if(affect_Y_N[index_fem] == "Y"){
            affect_Symbol = "Bolt_Male_Female";
}
else{
            affect_Symbol = "Bolt_Male";
}

// Print results-----------------------------------------------------------
System.out.println("The status of xlinked is: " + xlinked );
System.out.println("The  index  for  the  first  affected  man  is:  "  +
index_aff );
System.out.println("The color of his card is: " + code_Left[index_aff] );
System.out.println("The  index  for  the  first  woman  with  card  "   +
index_fem );
System.out.println("The  type  of  affection  symbol  is:  "  +
affect_Symbol );

} // end main method

} // end of class
```

The algorithmic structure of the procedure is sequential, as illustrated in Table I. The first task is to determine if we are in the presence of an x-linked case. If that is the case, then the next step is to identify a male with an affected phenotype. This is followed by a search of a female that shares the genotype of the affected male. Finally, based on the phenotype of the female selected, a decision is made on whether the gene is dominant or recessive.

Several conclusions can be derived from the analysis of the java program. The first conclusion is that the genetic inheritance data provided to the students is not optimally coded. Relevant pieces of information of the constituents of the problem are coded in separate graphs.

A set of six arrays in the java program represent the initial data provided in the problem. Potentially all data might be relevant.

The second conclusion is that the algorithm is a well-structured set of simple procedures that need to be implemented in the appropriate order. This order of implementation accesses the input data in a predictable form: each set of data is relevant at a particular step in the process.

It is difficult to precisely know why in the reference study only 20% of college students were able to correctly solve the problem after being lectured by the professor, but several hypotheses can be proposed. The most relevant hypothesis is that students were prepared and trained to solve the problem using a heuristic-search strategy. Having studied several examples of alternative forms of the problem, the students were expected to bring to the solution those experiences that were relevant to the case, and in a trial and error form, try to confirm or reject alternative solutions according to past experience. It is highly probable that no formal algorithmic solution was presented to the students. Finally, it is very likely that a great deal of cognitive process was dedicated to re-code the information provided into a form that would allow the resolution of the problem.

C. Isomorphic Problem: Card Game

The same java code created to solve the genetic inheritance problem was used to create a Card-game in which the participants were presented with a family portrait. The members of the family were described as having blue or red faces (phenotype), and holding color cards in one or two hands (genotype.)

The goal of the game was to identify the gene (color card) that was the cause of the affection, and whether the gene was dominant of recessive. Also, one goal of the game was to determine if it was an x-linked or autosomal mode of inheritance problem.

In the computational method developed for this experiment, both phenotype and genotype information were integrated into a single diagram (a blue/red face represented the phenotype, and one or two cards held in the hands of the human symbols represented the microsatellite genotype information. See Fig. 1 and Fig. 2).

Figure 1. Video tutorial integrated in google forms

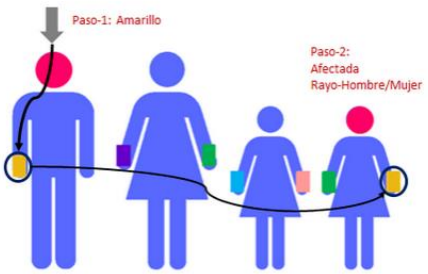

Figure 2. Illustrated solution during training

The data coding and algorithmic structure of the Card-game problem were designed using knowledge of human cognitive primitives. Evolution has endowed the human species with very powerful cognitive resources, or primitives. These primitives are in areas such as language processing, spatial navigation, face and mood recognition, object oriented thinking, generalization from samples, and many others. The Card-game problem was designed to be supported, both in data coding and in algorithmic representation, by a set of such cognitive primitives.

III. Materials and Methods

A. Samples

A set of experimental activities were designed to be implemented during the academic year 2014-15 in the classrooms of four Spanish universities, in particular in schools and departments of Education. The four participating universities are: the University of Alicante, the University of Extremadura, the University of the Basque Country, and the University of Salamanca. These universities belong to four separate and independent autonomic regions in Spain: Comunidad Valenciana, Extremadura, Pais Vasco, y Castilla Leon, respectively.

TABLE II. Students Statistics from Participating

	Sex		
University	Men	Women	Total
University of the Basque Country	7.00	29.00	36.00
	19.44%	80.56%	100.00%
	16.67%	19.33%	18.75%
	3.65%	15.10%	18.75%
University of Extremadura	4.00	39.00	43.00
	9.30%	90.70%	100.00%
	9.52%	26.00%	22.40%
	2.08%	20.31%	22.40%
University of Salamanca	17.00	44.00	61.00
	27.87%	72.13%	100.00%
	40.48%	29.33%	31.77%
	8.85%	22.92%	31.77%
University of Alicante	14.00	38.00	52.00
	26.92%	73.08%	100.00%
	33.33%	25.33%	27.08%
	7.29%	19.79%	27.08%
Total	42.00	150.00	192.00
	21.88%	78.13%	100.00%
	100.00%	100.00%	100.00%
	21.88%	78.13%	100.00%

Table II summarizes the percentage of women and men participating in the experiments grouped by university. The participants were freshmen and sophomore college students, 18-20 years of age.

B. Video Tutorial

The isomorphic problem, Card-game, was introduced to the students using a video-tutorial. The tutorial was designed to describe the representation of the data structures of the problem, and its main algorithmic steps. Once the theoretical concepts of the problem were introduced, the video tutorial illustrated these concepts with an example. The total duration of the tutorial was five minutes.

C. Methods

The experiences described in this section were implemented as part of a workshop with the title: "Experimental Workshop in the Classroom:

Computational Model of the Students' Cognitive Processes".

The workshop was implemented in the four participating universities on separate dates. The workshops were facilitated by a member of the research group with the assistance of the local professor in each campus. These sessions took place in multimedia equipped classrooms, with projector, audio system, and computer and internet access for each student.

For the delivery of the video tutorials and the collection of data, Google Forms were used. The participants were informed of the characteristics of the experiment and were asked to verify their consent of participation. The total duration of each experiment was 15 minutes.

The experience consisted of three phases:

- Phase 1.- Algorithmic Knowledge
- Phase 2.- Training-Test
- Phase 3.- Subjective Survey

In phase 1 the students viewed the 5 minute video tutorial. They had the ability to pause and rewind the video if necessary. After the viewing of the video, the participants responded to a set of eight multiple-choice questions evaluating their knowledge about the data representation and algorithmic process of the task. Fig. 1 shows a frame of the video tutorial.

Phase 2 was divided into two parts, training and test. During the training part, the participants responded to three problems by identifying two variables in each problem. After each problem the participants could evaluate their responses by comparing them to the correct answers. During the test part the participants needed to solve five problems. The participants did not receive any feedback on their performance during the test part. Fig. 2 illustrates an example of feedback with solution during the training session.

During phase 3, the participants provided personal and subjective information about cognitive aspects of the experience by numerically labeling their experience in nine areas. Table III lists the content of the subjective questions to be answered by the participants.

All analyses presented in this paper were implemented using the software package SPSS v. 19.

TABLE III. SUBJECTIVE ASPECTS OF THE EXPERIENCE

Easy_to_Learn	Did you find it easy to learn the concepts and protocols described in this session?
Easy_to_Teach	Do you think it would be easy for you to successfully teach another person what you learned in this session?
Learning Stress	Did you feel stressed while responding to the questions during the test?
Subject	In which of the following areas of knowledge do you find yourself at ease? (7 choices)
Course_Biology	Have you had any courses in Biology?
Solve_Math	How would you rate your ability to solve Mathematics problems?
Teach_Math	How would you rate your ability to teach Mathematics?
Solve_Biology	How would you rate your ability to solve problems in Biology?
Teach_Biology	How would you rate your interest in teaching Biology?

IV. RESULTS AND DISCUSSION

A. Results

In this section we present the results obtained in the three phases of the experience. These results will be presented in a quantitative mode.

Table IV presents data relative to the first phase of the experience. A total of eight questions were asked, with a total maximum score of 9 points. These questions were related to the data representation and the algorithmic structure of the studied problems. Six questions had a maximum score of 1 point, and two questions had a maximum score of 1.5 points.

The average score of the 192 participants was 8.54, with a standard deviation of 0.79.

TABLE IV. SCORE ON DATA AND ALGORITHMIC KNOWLEDGE

Assessment of Algorithmic Knowledge (0 to 9 points.)		
N	Valid	192
	Lost	0
Average		8.54
Std Dev		.79
Min		5.50
Max		9.00

The results of phase 2 are presented in Table V. In this phase the participants had to solve three problems in a training part, and five additional problems in a test part. In the training part the participants received feedback after they solved the problem. No feedback was provided during the five test questions. Each problem required the identification of two variables, therefore there were six questions in the training part, and ten questions in the test part (Tra_1 through Tra_6, and Test_1 through Test_10.)

The results show that all questions were correctly answered by the 192 participants at a rate of 93% or higher.

TABLE V. TRAINING AND TEST RESULTS

Training	**% success**		**Test**	**% success**
Tra_1	98.96		Test_1	94.27
Tra_2	94.79		Test_2	98.44
Tra_3	98.44		Test_3	93.23
Tra_4	97.40		Test_4	97.92
Tra_5	97.40		Test_5	94.79
Tra_6	96.35		Test_6	97.40
			Test_7	98.44
			Test_8	96.35
			Test_9	97.40
			Test_10	96.68

The results of phase 3 evaluate several aspects relevant to the experience.

An instrument was designed to quantify the 'pedagogical value' of the presented methodology in the

Card-game. For this purpose, two items were used: Easy_to_Learn and Easy_to_Teach (each item uses a Likert scale with nine levels.) The validity of the internal consistency of the instrument was determined by a Cronbach's α above the threshold 0.7 (α = 0.73). Table VI shows the results of the pedagogical value assessment.

TABLE VI. EVALUATION OF THE PEDAGOGICAL VALUE

Assessment of Pedagogical Value (1 to 9 points.)			
		Easy_to_Learn	Easy_to_Teach
N	Valid		192
	Lost		0
Average		8.10	7.47
Std Dev		1.58	1.62
Min		1.00	1.00
Max		9.00	9.00

An additional aspect analyzed in the experience is the existing relationship between the score in the 'algorithmic knowledge' and the university where the experience took place. The results of this analysis are summarized in Table VII.

TABLE VII. ALGORITHMIC KNOWLEDGE AND UNIVERSITY

	N	*Average*	*Standard Deviation*
University of the Basque Country	36	8.31	.98
University of Extremadura	43	8.64	.64
University of Salamanca	61	8.56	.81
University of Alicante	52	8.61	.74
Total	192	8.54	.79

There was not a statistically significant difference between groups as determined by one-way ANOVA ($F(3,188) = 1.41$, $p = .242$).

Finally, an analysis was implemented to study the relationship between the 'algorithmic knowledge' of the participants and their preference in area of study (*'Subject: Which of the following areas-of-knowledge do you find yourself more identified with?'*). The results are summarized in Table VIII.

TABLE VIII. ALGORITHMIC KNOWLEDGE AND AREA OF STUDY

	N	*Average*	*Standard Deviation*
Natural Sciences and Life Sciences	25	8.50	.82
Artistic Drawing and Plastic Arts	14	8.50	.78
Physics and Chemistry	2	7.75	1.77
Informatics and Technology	20	8.60	.70
Language and Foreign Languages	33	8.52	.76
Mathematics	35	8.61	.72
Psychology and Pedagogy	63	8.55	.87
Total	192	8.54	.79

There was not a statistically significant difference between groups as determined by one-way ANOVA ($F(6,185) = 0.41$, $p = .869$).

B. Discussion

The experiments described in this paper were implemented to address several questions:

- Is it possible to represent in a computational language the data structures and algorithmic strategies required to successfully solve Type-A problems, typically encountered by students in multiple-choice tests in secondary and undergraduate education?
- How could these data structures and algorithmic strategies be effectively coded using the human computational primitives and avoiding heuristic-search strategies?
- Does the choice of teaching methodology (heuristic search vs computational thinking) affect the performance of the general population in problems considered 'difficult' in the published literature?

The experiences described in this paper indicate that, at least in areas similar to that explored here: it is possible to formalize a computational solution to a Type-A problem; there are strategies that benefit from the existing cognitive primitives of humans; and there is a significant degree of success when students utilize a computational paradigm to solve isomorphic problems.

One of the goals of the work presented here is to open areas of research that will address the following topics:

- Use of information theory techniques to determine the cognitive difficulty of a problem based on computational parameters and not based on traditional failure rates of students.
- Study the existing evolutionary cognitive primitives of the human brain and their integration in the resolution of complex problems.
- Exploration of areas of knowledge where the efficient use of these human capabilities would bring important progress to society.
- Study of educational methodologies that could be developed using relevant findings from these studies.

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