

Method and Verification of a Supporting Application for the Unfolded Version of the Rubik's Cube

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Abstract—One approach to mathematics education conceptualizes mathematics as a form of game. Extensive research in mathematics education and pedagogy has examined game-based learning, focusing on instructional strategies and learners' cognitive processes. This approach requires relatively little prior mathematical knowledge, making it accessible to a wide range of learners and relatively easy for instructors to implement. In recent years, a new unit entitled "Mathematics in Games and Puzzles" has been introduced into the Japanese high school mathematics curriculum. This educational framework aims to deepen students' mathematical understanding through games and puzzles in a relaxed learning environment. In response, a variety of teaching materials and instructional designs have been developed, and their educational effectiveness has been examined from the perspectives of interdisciplinary collaboration and relevance to Science, Technology, Engineering, Arts, and Mathematics (STEAM) education. Among such games, the Rubik's Cube is a representative example and can be used as teaching material for fostering and assessing mathematical thinking skills. Furthermore, digital versions of the Rubik's Cube have been proposed to enable the recording and analysis of manipulation processes. However, solving the Rubik's Cube using planar representations in the form of geometric nets can impose a greater cognitive load on learners. In this study, we focus on an application that supports operations on unfolded representations of the Rubik's Cube, propose a framework for data acquisition and methods for representing problems and solution processes, and examine the effectiveness of the proposed approach.

Keywords—Rubik's Cube, unfolded representation, educational application, operation log, mathematics education, STEAM

I. INTRODUCTION

Game-based approaches to mathematics education have long been practiced and recognized as an effective means of fostering engagement and logical reasoning among students. In this pedagogical approach, games are not merely viewed as recreational activities but as structured environments where mathematical ideas and strategic thinking can be explored in a tangible manner. Research on mathematical methods for developing winning strategies has been conducted through educational investigations into simple yet deeply mathematical games such as rock-paper-scissors [1]. These studies have demonstrated that even games governed by chance or simple rules can reveal profound insights into probability, strategy, and decision theory. Likewise, mathematical analyses have been applied to more complex games involving playing cards, which allow for the exploration of combinatorics, probability distributions, and even game theory in accessible classroom contexts [2].

In contemporary education, increasing emphasis has been placed on identifying and utilizing the mathematical structures underlying everyday games and puzzles. This perspective has led to the introduction of a new unit in the Japanese secondary school mathematics curriculum titled "Mathematics in Games and Puzzles". The primary aim of this initiative is to make mathematical learning more engaging, interactive, and accessible for younger learners who may otherwise perceive mathematics as abstract or disconnected from daily experience. To achieve this goal, approaches that encourage logical reasoning, problem decomposition, and strategic exploration through games and puzzles have been actively promoted. Examples include the development and implementation of

instructional materials based on card games, educational practices that draw connections between stone-taking games and mathematical reasoning [3], and classroom activities using models derived from the Tower of Hanoi puzzle [4]. Collectively, these practices contribute to cultivating mathematical thinking through play-based learning, blending cognitive development with experiential exploration.

Among the most widely studied examples of such mathematical puzzles is the Rubik's Cube. Since its invention, the Rubik's Cube has not only been regarded as a challenging recreational puzzle but has also been recognized as an educational tool that promotes spatial reasoning and algorithmic thinking. Prior research has examined a variety of topics, including solving algorithms for different cube sizes [5], methodologies for recording and representing the solving process [6], and the development of digital systems for supporting problem-solving and algorithmic exploration in mathematics education [7]. Although numerous studies have been conducted on the Rubik's Cube, even the standard 3×3 version presents an enormous number of possible configurations—over 43 quintillion states—making it computationally complex and cognitively demanding. To address this challenge, the present study focuses first on the 2×2 version, which can be considered a lower-dimensional model that retains the cube's logical structure while simplifying the analysis.

As illustrated in Fig. 1(a), even a Rubik's Cube with a 2×2 configuration possesses a vast array of possible move sequences, rendering brute-force solution methods infeasible. Therefore, theoretical analysis, classification of rotational operations, and the systematic study of state transitions are regarded as effective approaches in mathematics education. These analytical approaches not only reveal underlying mathematical patterns but also encourage students to connect abstract reasoning with physical manipulation. The Rubik's Cube, composed of six faces divided into smaller colored squares, requires the solver to restore each face to a uniform color by a series of rotations. Solving such a puzzle fosters critical thinking, combinatorial reasoning, and an appreciation of algorithmic efficiency—key aspects of mathematical thought [8].

Recognizing this educational potential, researchers have proposed various electronic versions of the Rubik's Cube, such as that shown in Fig. 1(b) [9]. These systems automatically record the user's operation sequences and utilize these logs within a 3D virtual environment to evaluate problem difficulty, generate new learning tasks, and explore performance assessment methods [10]. Further studies conducted with student participants have proposed visualization techniques that map the relationship between a user's solving process and the minimum number of moves required to reach the goal configuration. Through such analyses, it has been possible to evaluate problem-solving efficiency and cognitive strategy. Additionally, experimental studies have examined the cognitive processes involved in tasks requiring only a few moves, providing insight into how

learners internalize spatial transformations [11]. Among the various proposed assessment approaches, one particularly interesting method focuses on evaluating problem-solving through the geometric center of gravity of the cube's configuration, providing a quantitative perspective on the solving process [12].

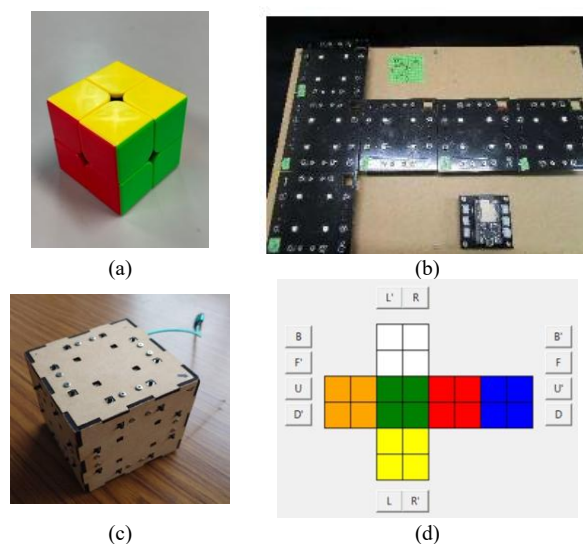


Fig. 1. Rubik's Cube-related research: (a) regular type; (b) electronic type [10]; (c) expanded type; (d) software-based type.

Subsequent research has also explored algorithmic optimization methods to improve computational efficiency in real-time data acquisition systems designed for problem-solving support [13]. For visual representation, Python-based visualization tools have been developed to render the cube's six 3D faces dynamically, enabling both real-time monitoring and interactive manipulation. This digital implementation allows for the cube to be treated as a geometric net version, where the 3D structure can be unfolded into a 2D representation for educational use [14]. Such representations help bridge the gap between spatial reasoning and planar geometric understanding, making abstract cube manipulations more comprehensible to learners.

Building upon these advances, an application-based version of the Rubik's Cube was developed, as shown in Fig. 1(c). However, user feedback—particularly from experienced solvers—indicated that this version was significantly more challenging than its 3D counterpart. Participants noted that it required advanced spatial reasoning skills and complex mental rotation, which hindered its accessibility for general learners [15]. To address these challenges and reduce the cognitive load associated with visual-spatial transformations, we developed an enhanced application featuring the unfolded version of the Rubik's Cube, as shown in Fig. 1(d) [16]. This version enables users to manipulate the cube both as a 3D object and as a 2D unfolded representation, facilitating an intuitive understanding of spatial correspondences.

In our previous research, we introduced this application, verified its functionality through pilot testing, and confirmed its educational potential. However, we did not

yet formulate the full set of problems and corresponding solutions required to utilize the unfolded version of the Rubik's Cube effectively within a structured learning environment. In the present study, we address this limitation by systematically defining and validating these problem sets, thereby enhancing the educational value and experimental rigor of the system.

Specifically, to effectively utilize the proposed unfolded net-based Rubik's Cube application [16], we formalize the problems and their corresponding solution methods. Furthermore, experiments involving both beginners and experienced participants are conducted, and their sequences of operations are recorded and analyzed to examine the differences between the two groups.

The remainder of this paper is organized as follows. Section II outlines the framework of the electronic Rubik's Cube and the systems developed to date, and describes the details of the proposed educational support application. Section III presents the experimental results and discussion obtained using the developed educational tool, and provides an analysis based on a comparison of the performance of beginners and experienced participants. Finally, Section IV summarizes the findings of this study and discusses the conclusions and directions for future work.

II. RUBIK'S CUBE APPLICATION AND PROBLEM SETTING

This section provides an overview of the application for the unfolded version of the Rubik's Cube and presents the formulation necessary for problem setting.

A. Application Overview

A PC application was developed using Python [16] to support Rubik's Cube operations. Several rotation buttons ① are arranged around the unfolded diagram, as shown in Fig. 2. The interface includes 12 types of buttons, each corresponding to a specific move: F , B , L , R , U , D , F' , B' , L' , R' , U' , or D' . Users perform cube manipulations by pressing the appropriate button for each move, enabling intuitive interaction with the unfolded version of the Rubik's Cube representation.

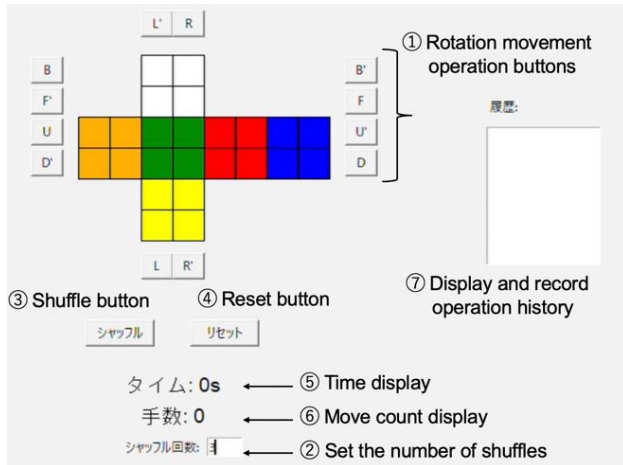


Fig. 2. Rubik's Cube application.

The number of shuffles can be entered from the

keyboard. Specify the number of shuffles ②. A shuffle button ③ and a reset button ④ are also arranged, and pressing the shuffle button activates the time display ⑤ below it. There is also a display ⑥ that counts the number of moves required to match the color of the entire surface. A history of the names of the rotation buttons pressed so far is displayed on the right side ⑦.

B. Display Method for Problem Setting and Solution

In this section, we mathematically formulate the problems and their solutions required for problem setting. The six faces of the 2^3 -ERC are represented as (O, G, R, B, Y, W), where O, G, R, B, Y, and W correspond to orange, green, red, blue, yellow, and white, respectively. In addition, the remaining four faces are assigned the partitions 1, 2, 3, and 4 (see Fig. 3).

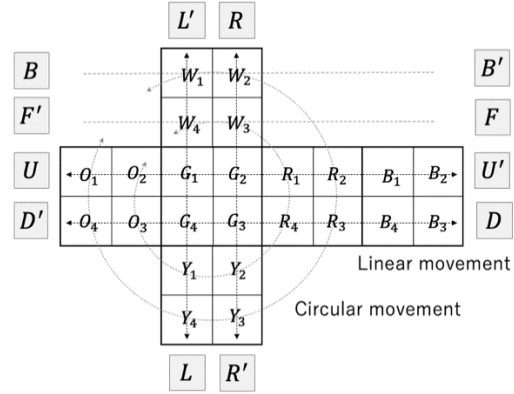


Fig. 3. Method for moving surface elements in the unfolded Rubik's Cube.

Therefore, the state of the unfolded drawing can be expressed by defining a 24-dimensional vector q containing a total of 24 elements as follows:

$$q = \begin{pmatrix} O_1, O_2, O_3, O_4, G_1, G_2, G_3, G_4, R_1, R_2, R_3, R_4, \\ B_1, B_2, B_3, B_4, Y_1, Y_2, Y_3, Y_4, W_1, W_2, W_3, W_4 \end{pmatrix} \in \mathbb{R}^{24} \quad (1)$$

The top surface of the unfolded version of the Rubik's Cube moves linearly using the eight buttons L , R , U , D , L' , R' , U' and D' . The corresponding mappings for these are defined as f_L , f_R , f_U , f_D , $f_{L'}$, $f_{R'}$, $f_{U'}$ and $f_{D'}$.

The four buttons F , B , F' , and B' move circularly. The corresponding mappings for these are defined as f_F , f_B , $f_{F'}$, and $f_{B'}$. Therefore, the following holds: $f_L(q)$, $f_R(q)$, $f_U(q)$, $f_D(q)$, $f_{L'}(q)$, $f_{R'}(q)$, $f_{U'}(q)$, $f_{D'}(q)$, $f_F(q)$, $f_B(q)$, $f_{F'}(q)$, $f_{B'}(q) \in \mathbb{R}^{24}$.

Therefore, the relation (2) for the next state can be derived from the current state using the mapping defined below.

$$f: q \in \mathbb{R}^{24} \rightarrow f(q) \in \mathbb{R}^{24} \quad (2)$$

Also, since f is bijective, there exists an inverse mapping f^{-1} as follows:

$$f^{-1}: f(q) \in \mathbb{R}^{24} \rightarrow q \in \mathbb{R}^{24} \quad (3)$$

From the above, any problem can be expressed as a product of repeated permutational mappings arbitrarily chosen from the defined operations. Furthermore, by using Eq. (3), the answer to the problem can be expressed as a product of the corresponding inverse permutational mappings.

III. EXPERIMENT

In the experiment, problems with a predetermined number of moves were prepared in advance. The experiment was conducted at a technical college enrolling students aged 16 to 20, and the participants consisted of two students with prior experience using the unfolded version of the Rubik's Cube and two students without such experience. Section A describes the experimental setup, Sections B and C present the experimental results in detail, and Section D provides discussion and concluding remarks.

A. Problem Setup

The problem setup was done without using the application's automatic generation function (shuffle function) and was based on the initial state only, as shown in Fig. 4(a). The teacher set the problems by pressing the 12 buttons on the app: $F, B, L, R, U, D, F', B', L', R', U', D'$ (Fig. 4(b)–(f)).

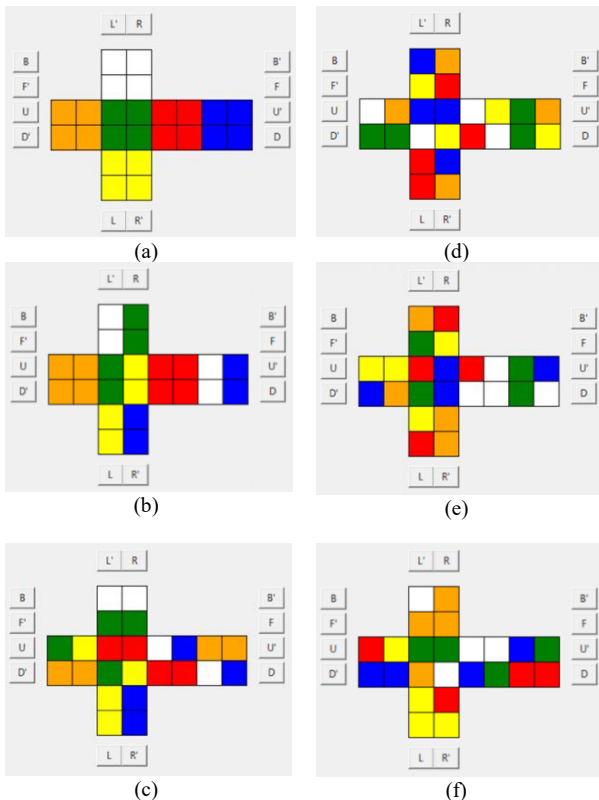


Fig.4. Problem setup: (a) Initial state; (b) 1-move problem; (c) 2-move problem; (d) 3-move problem; (e) 4-move problem; (f) 5-move problem.

The actual problems and answers are shown in Table I using (2) and (3). The five problems were defined as Problem I (1-move problem), Problem II (2-move problem), Problem III (3-move problem), Problem IV

(4-move problem), and Problem V (5-move problem).

TABLE I. SET UP FOR PROBLEMS AND ANSWERS

Number of moves	Initial status	Answer
1	f_R	f_R^{-1}
2	$f_U f_R$	$f_R^{-1} f_U^{-1}$
3	$f_U f_R f_F$	$f_F^{-1} f_R^{-1} f_U^{-1}$
4	$f_R f_B' f_U f_R$	$f_R^{-1} f_U^{-1} f_B^{-1} f_R^{-1}$
5	$f_U' f_R' f_U f_D f_F$	$f_F^{-1} f_D^{-1} f_U^{-1} f_R^{-1} f_U^{-1}$

B. Experimental Result 1

The presented tasks were conducted with participants S1 and S2. Both participants had prior experience in manipulating the 3D version of the Rubik's Cube. Tables II and III show the operation time for the three-dimensional Rubik's Cube as t_c , the operation time for the unfolded version of the Rubik's Cube as t_p , and the number of operations as n . When a participant failed to reach a solution to a given problem, "GU" (Give Up) is indicated. The time limit was set to 5 minutes, and the application was configured to terminate automatically when the time limit was reached.

As shown in Table II, for participant S1, the operation time t_c for the 3D version of the Rubik's Cube increases as the problem number increases. For the unfolded version of the Rubik's Cube, the operation time t_p also increases with increasing problem number.

As shown in Table III, for participant S2, the operation time t_c for the 3D version of the Rubik's Cube increases as the problem number increases. For the unfolded version of the Rubik's Cube, both the operation time t_p and the number of steps n increase with increasing problem number.

TABLE II. STUDENT S1 (EXPERIENCED)

Problem number	3D version	Unfolded version	
	Time: t_c	Time: t_p	n
1	00:00:05	00:01:04	1
2	00:02:29	00:03:00	2
3	00:06:17	00:03:81	3
4	00:01:10	00:11:62	4
5	00:31:25	00:14:18	5

TABLE III. STUDENT S2 (EXPERIENCED)

Problem number	3D version	Unfolded version	
	Time: t_c	Time: t_p	n
1	00:01:00	00:01:05	1
2	00:03:08	00:10:03	2
3	00:10:08	02:14:02	37
4	00:39:08	03:49:03	130
5	00:01:08	06:23:07	329

C. Experimental Result 2

Next, we present the experimental results obtained from participants with no prior experience in manipulating a Rubik's Cube. Tables IV and V show, for participants D1 and D2, the operation time for the 3D version of the Rubik's Cube, the operation time for the unfolded version of the Rubik's Cube, and the number of operation steps.

As shown in Table IV, for participant D1, the operation

time t_c for the 3D version of the Rubik's Cube increases as the problem number increases. For the unfolded version of the Rubik's Cube, both the operation time t_p and the number of steps n increase with increasing problem number. In addition, for Problem Numbers 4 and 5, the participant Gave Up (GU).

As shown in Table V, for participant D2, it cannot be said that the operation time for the 3D version of the Rubik's Cube increases with increasing problem number. For the unfolded version of the Rubik's Cube, the operation time increases as the problem number increases. Furthermore, for Problems 3 and 5, the participant GU.

TABLE IV. STUDENT D1 (INEXPERIENCED)

Problem number	3D version	Unfolded version	
	Time: t_c	Time: t_p	n
1	00:01:56	00:02:24	1
2	00:08:31	00:11:12	2
3	81.22 GU	00:17:07	3
4	01:19:58 GU	03:00:00 GU	54
5	01:26:33 GU	03:00:00 GU	67

TABLE V. STUDENT D2 (INEXPERIENCED)

Problem number	3D version	Unfolded version	
	Time: t_c	Time: t_p	n
1	00:01:32	00:05:77	1
2	00:02:09	00:18:00	10
3	00:45:00 GU	00:43:08	13
4	00:22:48	01:14:66	18
5	00:47:00 GU	03:00:00 GU	56

D. Comparing the Results of Experienced and Inexperienced Participants

As a result of comparing the two groups, the following findings were obtained.

In both the three-dimensional and unfolded net-based Rubik's Cube tasks, experienced participants generally completed more of the assigned tasks, whereas some inexperienced participants gave up before completion. Therefore, direct comparisons of operation time and number of steps between the two groups should be interpreted with caution.

Experienced participants achieved a 100% correct answer rate on both the three-dimensional and unfolded version problems. In contrast, the correct answer rate for inexperienced participants was 50% on the 3D version problems and 70% on the unfolded version problems.

For the inexperienced participants, the correct-answer rate for the unfolded version (70%) was higher than that for the 3D version (50%). Under the experimental conditions of this small-sample study, the experienced participants completed all tasks in both formats, whereas the inexperienced participants did not. These results should be interpreted as preliminary because of the limited number of participants.

IV. CONCLUSION

In this study, we developed a problem-setting methodology for an application that supports manipulation of the unfolded version of the Rubik's Cube. Tasks with

predetermined numbers of moves were presented to experienced and inexperienced participants, and their operation processes were recorded and analyzed. The experienced participants completed all tasks in both the 3D and unfolded formats, whereas some inexperienced participants gave up before completing the more difficult tasks. For the inexperienced participants, the correct-answer rate was 50% for the 3D version and 70% for the unfolded version. These results demonstrate the feasibility of using the proposed application to present structured tasks and record solution processes; however, because the experiment involved only four participants, further studies with a larger sample are required to evaluate learning effects and differences attributable to prior experience.

CONFLICT OF INTEREST

The authors declare no conflict of interest.

AUTHOR CONTRIBUTIONS

K.K. and K.J. conducted the research, collected the data, performed the data analysis, and wrote the manuscript. A.N., T.U., D.P., and K.M. contributed ideas for the analytical methodology and provided scientific guidance. All authors reviewed and approved the final version of the manuscript.

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